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Nonlinear Smoothing Identification Algorithm with Application to Data Consistency Checks

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A parameter identification algorithm for nonlinear systems is presented. It is based on smoothing test data with successively improved sets of model parameters. The smoothing, which is iterative, provides all of the information needed to compute the gradients of the smoothing performance measure with respect to the parameters. The parameters are updated using a quasi-Newton procedure, until convergence is achieved. The advantage of this algorithm over standard maximum likelihood identification algorithms is the computational savings in calculating the gradient. This algorithm was used for flight-test data consistency checks based on a nonlinear model of aircraft kinematics. Measurement biases and scale factors were identified. The advantages of the presented algorithm and model are discussed.

Nomenclature

a_x, a_y, a_z = body axis components of the translational acceleration
$a_{x_{cg}}, a_{y_{cg}}, a_{z_{cg}}$ = body axis components of the translational acceleration at the center of gravity
b = measurement bias vector, dimension m
b_ξ = measurement bias
$d_{(\cdot)}$ = process noise input in the SMACK model
E = $m \times m$ diagonal matrix of measurement scale factors
f_c = nonlinear continuous dynamics vector function, dimension n
f_d = nonlinear discrete dynamics vector function, dimension n
f_x, f_w = gradient matrices of the vector function f_c
h = altitude
h_m = nonlinear measurement vector function, dimension m
h_x = gradient matrix of the vector function h_m
J = performance measure
L = transformation matrix from Earth to aircraft body coordinate system
L_p, L_{δ_a} = aircraft stability and control derivatives
$\ell_{\xi_x}, \ell_{\xi_y}, \ell_{\xi_z}$ = sensor location measured from the center of gravity in the aircraft body coordinates
n_ξ = measurement noise
P_0 = initial conditions weighting matrix
p, q, r = body axis components of the angular rate
Q = process noise weighting matrix
R = output error weighting matrix
$\mathcal{R}, \mathcal{B}, \mathcal{E}$ = ground track data: slant range, bearing, and elevation angles
s_ξ = measurement scale factor
T = transformation matrix from aircraft body to Earth coordinate system
t_i = time at the sampling instants, such that $t_i = t_0 + i\Delta t$ for $i = 1, \ldots, N$ and Δt is the time between samples
u = known input vector, dimension c
u_a, v_a, w_a = body axis components of the translational velocity at the center of gravity assuming calm atmosphere
u_{cg}, v_{cg}, w_{cg} = body axis components of the translational velocity at the center of gravity
V = total airspeed
v = measurement noise vector, dimension m
w = process noise vector, dimension q
x = state vector, dimension n
x_e, y_e = aircraft north and east coordinates relative to a reference point (e.g., tracking radar location)
x_0 = a priori estimate of the state initial conditions $x(0)$
z = measurement vector, dimension m
$z_m(t_i)$ = experimental data, actual measurements
α = angle of attack
β = sideslip angle
δ_a = aileron deflection angle
Θ = parameters weighting matrix
θ = vector of model parameters, dimension p
θ_0 = a priori estimate of the parameters θ
$\bar{\theta}$ = extended parameter vector that includes the parameters θ, the scale factors in E, and the biases b
$\bar{\theta}_0$ = a priori estimate of the parameters $\bar{\theta}$
λ = Lagrange multiplier
$\nu(t_i)$ = difference betwen the measured outputs and the model outputs $z_m(t_i) - z(t_i)$
ξ, ξ_m = a generic system output and its measurement
ϕ, θ, ψ = body attitude angles: roll, pitch, and yaw

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I. Introduction

ACCURATE experimental test data are required in many engineering applications such as system modeling, validation of simulation codes, and evaluation of the system performance. In aeronautical applications it may include flight envelope investigations,[1] mathematical modeling of aircraft aerodynamics,[2] and validation of flight simulators.[3] To assure this accuracy, the experimental data are checked for consistency using some known relationships among the redundant measurements. If the data are not consistent, identification procedures must be used to resolve these discrepancies. Procedures are also needed to reconstruct important states of the system that were not measured.

In practice, the aircraft states have to be reconstructed from a set of measurements that may contain significant errors, e.g., measurement scale factors and biases and random measurement noise. The measurements are linear and nonlinear functions of the states. Usually not all of the aircraft states are measured directly, and the goal is to determine a complete and consistent set of aircraft state histories from the measurements by simultaneously identifying the measurement scale factors, biases, and random errors.

Kinematic equations can be used to reconstruct a consistent set of aircraft data from redundant measurements. These equations consist of nonlinear dynamic relationships among the various states of the aircraft. Thus, to identify the measurement scale factors and bias errors, a parameter identification algorithm for nonlinear systems is required. In this paper, an algorithm, based on smoothing, is presented and used for checking flight-test data consistency.

The parameter identification task can be defined as follows: Given a system model that contains constant parameters, determine these parameters so that the response of the model to a given set of inputs best fits the experimental system response to the same input signals. The quality of the fit is measured by a scalar function of the difference between the experimental output data and the model output, called the performance measure. A commonly used performance measure is a weighted least squares function of this difference.

Maximum likelihood estimation has become a standard approach for parameter identification, widely used in many fields of engineering, e.g., aircraft dynamic modeling from flight test data.[2,4,5] Most of the existing algorithms are based on filtering the experimental data to account for the effects of process and measurement noises present in the model and on computation of the parameter sensitivity functions. Alternatively, the model parameters can be treated as additional constant states and a nonlinear filtering algorithm, such as the extended Kalman filter,[6,7] or a nonlinear smoothing algorithm[8,9] can be used.

A maximum likelihood type identification algorithm that is based on smoothing for linear systems was presented recently.[10-12] In this approach, the experimental data are smoothed using successively improved sets of system parameters. As part of the smoothing solution, the adjoint variables or the sensitivities of the performance measure to the states of the system are computed. These sensitivities are used to compute the gradients of the performance measure with respect to the unknown parameters. The parameters are then updated using a quasi-Newton gradient algorithm. The data are then smoothed with the new set of parameters. This procedure is repeated until a minimum of the performance measure is obtained. The advantages of the smoothing identification algorithm is in the computation of the gradient.[11-13]

In this paper, an identification algorithm for nonlinear systems is derived, based on the smoothing and identification approach mentioned earlier. This algorithm, together with a nonlinear kinematic model of the aircraft motion, is then used to identify the measurement scale factors and biases of actual flight-test data. The smoothed data that are corrected for the measurement errors can then be used for further analysis such as modeling of the aircraft aerodynamics.

First, the smoothing identification algorithm for nonlinear systems is derived. The kinematic model used for data consistency is then presented together with an alternative formulation. The two formulations are then compared. Results of flight-test data consistency checks and the reconstruction of the aircraft states are then presented. Concluding remarks are given at the end of the paper.

II. Identification Algorithm for Nonlinear Systems

Identification is the process of selecting parameters of a model to minimize some measure of the difference between the response of the model to a given set of inputs and the response of the actual system to the same input signals. A nonlinear system model can be described by state-space and measurement equations of the form:

$$\dot{x}(t) = f_c\left[x(t), u(t), w(t), t, \theta\right] \tag{1}$$

$$z(t_i) = Eh_m\left[x(t_i), u(t_i), t_i, \theta\right] + b + v(t_i) \tag{2}$$

The performance measure is usually a weighted least squares error. The identification task is to find the parameters θ, scale factors E, biases b, initial conditions $x(t_0)$, and the process noise inputs $w(t)$, $t_0 \leq t \leq t_f$ that minimize this performance measure:

$$J = \tfrac{1}{2}(\bar{\theta}-\bar{\theta}_0)^T\Theta^{-1}(\bar{\theta}-\bar{\theta}_0) + \tfrac{1}{2}\left[x(t_0)-x_0\right]^T P_0^{-1}\left[x(t_0)-x_0\right] + \frac{1}{2}\int_{t_0}^{t_f} w^T(t)Q^{-1}w(t)\,\mathrm{d}t + \frac{1}{2}\sum_{i=1}^{N} \nu^T(t_i)R^{-1}\nu(t_i) \tag{3}$$

The first term in Eq. (3) allows for the inclusion of prior knowledge of the parameters. For example, in the case of aircraft model identification from flight-test data, the prior information can include wind-tunnel data.

To minimize J subject to the constraints (1) and (2), the performance measure J is modified by adjoining these equations using the Lagrange multipliers $\lambda(t)$:

$$\bar{J} = J + \int_{t_0}^{t_f} \lambda^T(t)\left\{f_c\left[x(t), u(t), w(t), t, \theta\right] - \dot{x}(t)\right\}\mathrm{d}t \tag{4}$$

Minimizing $\bar{J}$ is equivalent to minimizing J subject to constraints (1) and (2).

Variational techniques are used to minimize $\bar{J}$. The first variation of $\bar{J}$ due to small changes of the unknowns of the problem is given by

$$\delta\bar{J} = \int_{t_0}^{t_f}\left[\frac{\partial\bar{J}}{\partial x(t)}\delta x(t) + \frac{\partial\bar{J}}{\partial w(t)}\delta w(t)\right]\mathrm{d}t + \sum_{i=0}^{N}\frac{\partial\bar{J}}{\partial x(t_i)}\delta x(t_i) + \frac{\partial\bar{J}}{\partial\bar{\theta}}\delta\bar{\theta} \tag{5}$$

At a stationary point of $\bar{J}$, the first variation $\delta\bar{J}$ vanishes for arbitrary variations of $\delta x(t_0)$, $\delta w(t)$, and $\delta\bar{\theta}$. For a given set of parameters $\bar{\theta}$, the previous describes a nonlinear smoothing problem. The derivation of a nonlinear smoothing algorithm that solves this problem is presented in Appendix A.

Once the smoothing problem is solved, it follows from Eq. (5) that

$$\delta\bar{J} = \frac{\partial\bar{J}}{\partial\bar{\theta}}\delta\bar{\theta} \equiv \bar{J}_\theta\delta\bar{\theta}$$

which also has to vanish at the stationary point. In general, that will not be the case and the parameters $\bar{\theta}$ must be changed to decrease the performance measure $\bar{J}$. Using the smoothed variables $\hat{x}(t)$, $\hat{w}(t)$, and $\hat{\lambda}(t)$, the gradient of $\bar{J}$ with respect to the parameters $\bar{\theta}$ can be computed by one of the following equations:

$$\frac{\partial\bar{J}}{\partial\theta(i)} = \sum_{j=1}^{p}\frac{\theta(j)-\theta_0(j)}{\Theta_\theta(i,j)} - \sum_{k=1}^{N}\hat{\nu}^T(t_k)R^{-1}E\frac{\partial h_m\left[\hat{x}(t_k), u(t_k), t_k, \theta\right]}{\partial\theta(i)} + \int_{t_0}^{t_f}\hat{\lambda}^T(t)\frac{\partial f_c\left[\hat{x}(t), u(t), \hat{w}(t), t, \theta\right]}{\partial\theta(i)}\mathrm{d}t \quad \text{for } i = 1,\ldots,p \tag{6}$$

$$\frac{\partial\bar{J}}{\partial E(i,i)} = \frac{E(i,i)-E_0(i,i)}{\Theta_E(i,i)} - \sum_{k=1}^{N}\hat{\nu}^T(t_k)R^{-1}\bar{E}_i h_m\left[\hat{x}(t_k), u(t_k), t_k, \theta\right] \quad \text{for } i = 1,\ldots,m \tag{7}$$

$$\frac{\partial\bar{J}}{\partial b(i)} = \frac{b(i)-b_0(i)}{\Theta_b(i,i)} - \sum_{k=1}^{N}\hat{\nu}^T(t_k)R^{-1}e_i \quad \text{for } i = 1,\ldots,m \tag{8}$$

where $\bar{E}_i$ is an $m \times m$ selector matrix and e_i is an m dimensional ith unit vector such that

$$\bar{E}_i(j,k) = \begin{cases} 1 & \text{if } j = k = i \\ 0 & \text{otherwise} \end{cases} \qquad e_i(j) = \begin{cases} 1 & \text{if } j = 1 \\ 0 & \text{otherwise} \end{cases}$$

The weighting matrix Θ is assumed to be a block diagonal matrix with blocks Θ_θ, Θ_E, and Θ_b. The matrices Θ_E and Θ_b are assumed to be diagonal.

From Eqs. (6-8) it is evident that the solution of the nonlinear smoothing problem provides *all* of the information required to compute the gradients of performance measure $\bar{J}$ with respect to *any* number of parameters in $\bar{\theta}$. To evaluate the integral in the last term of Eq. (6), the time histories of $\hat{x}(t)$, $u(t)$, $\hat{w}(t)$, and $\hat{\lambda}(t)$ between samples have to be approximated, since the measured control inputs $u(t_i)$ and the variables $\hat{x}(t_i)$, $\hat{w}(t_i)$, and $\hat{\lambda}(t_i)$ obtained from the discrete nonlinear smoothing algorithm are specified at the sample times t_i only (see Appendix A). Similar to the zero order hold (ZOH) assumption of the control inputs used to solve the nonlinear smoothing problem, $\hat{x}(t)$, $\hat{w}(t)$, and $\hat{\lambda}(t)$ are assumed to be constant between the samples. Using this ZOH approximation, the integral and thus the gradients can be evaluated. These gradients are then used to update the parameters $\bar{\theta}$ by a quasi-Newton procedure:

$$\bar{\theta}_{\text{new}} = \bar{\theta}_{\text{old}} - (\bar{J}_{\theta\theta})^{-1}\bar{J}_\theta \tag{9}$$

to minimize $\bar{J}$. The inverse of the Hessian matrix $(\bar{J}_{\theta\theta})^{-1}$, which is difficult to compute, will be estimated using the rank-two update procedure[14,15] presented in Appendix B. Smoothing with the new set of parameters, $\bar{\theta}_{\text{new}}$, is performed again, followed by a parameter update. This procedure is repeated until convergence is obtained and a minimum of J is reached.

To summarize, the outline of the nonlinear identification algorithm is as follows:

1) For a set of parameters $\bar{\theta}$, obtained from the preceding iteration or an initial guess $\bar{\theta}_0$, solve the associated nonlinear smoothing problem to compute the time histories of the smoothed states and the forcing functions, using a nonlinear smoothing algorithm (see Appendix A) and evaluate the performance measure J from Eq. (3).

2) Evaluate the gradient of $\bar{J}$ with respect to the parameters $\bar{\theta}$, using Eqs. (6-8).

3) Update the parameters $\bar{\theta}$ using a quasi-Newton procedure, where the inverse of the Hessian of $\bar{J}$ is estimated using the rank-two update algorithm (see Appendix B).

4) Repeat until the changes in $\bar{\theta}$ are "small" and the performance measure is minimized.

III. Aircraft Kinematic Model for Flight-Test Data Consistency Check

High-quality flight-test data are required for applications such as reliable modeling of aircraft aerodynamics[1] or helicopter rotor model validation.[16] For that, the data can be checked for consistency using the well-known kinematic relationships among the redundant measurements. Kinematic consistency of the data is assured by the identification of measurement scale factors and bias errors. In this work, the nonlinear smoothing and identification algorithm was used to identify these errors and to reconstruct the unmeasured states of the aircraft needed for further modeling.

A. Kinematic Model

The kinematic equations describing the motion of a rigid body with respect to a flat nonrotating Earth are

$$\begin{aligned} \dot{\phi} &= p + q \sin\phi \tan\theta + r \cos\phi \tan\theta \\ \dot{\theta} &= q \cos\phi - r \sin\phi \\ \dot{\psi} &= (q \sin\phi + r \cos\phi)/\cos\theta \end{aligned} \tag{10}$$

$$\begin{bmatrix} \ddot{x}_e \\ \ddot{y}_e \\ \ddot{h} \end{bmatrix} = T \begin{bmatrix} a_{x_{cg}} \\ a_{y_{cg}} \\ a_{z_{cg}} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \tag{11}$$

where

$$T = \begin{bmatrix} \cos\theta\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \cos\theta\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ \sin\theta & -\sin\phi\cos\theta & -\cos\phi\cos\theta \end{bmatrix}$$

If measurements of the three angular rates p, q, and r and the three translational accelerations $a_{x_{cg}}$, $a_{y_{cg}}$, and $a_{z_{cg}}$ are available, integration of Eqs. (10) and (11) should be consistent with the measurements of the states of the system $(\phi,\theta,\psi,x_e,y_e,h)$. However, due to measurement errors they will not be consistent.

A simple measurement model that describes the sensor inaccuracies includes measurement scale factors, biases, and additive random sensor noises. This model relates the "correct" outputs ξ to the actual measurements ξ_m by

$$\xi_m = s_\xi \xi + b_\xi + n_\xi \tag{12}$$

where s_ξ, b_ξ, and n_ξ are the scale factor, the bias, and the random noise, respectively. Equation (12) can be modified to include the effects of time delays in the measurements.[17] Often these delays are small, especially for measurements in the low-frequency range of the rigid-body motions and thus are not treated here.

The parameters in Eq. (12) can be identified using the kinematic relations (10) and (11) to generate a consistent set of data. For that, the measurements are divided into two groups: the angular rate and the translational acceleration measurements are treated as the "inputs" or "controls" of the kinematic model; the rest of the measurements are the "outputs" of the model. The "correct" rates and accelerations in the kinematic equations (10) and (11) are computed from the measured data by inverting Eq. (12):

$$\xi = \bar{s}_\xi \xi_m + \bar{b}_\xi + \bar{n}_\xi \tag{13}$$

where

$$\bar{s}_\xi = \frac{1}{s_\xi} \qquad \bar{b}_\xi = -\frac{b_\xi}{s_\xi} \qquad \bar{n}_\xi = -\frac{1}{s_\xi} n_\xi$$

These relationships together with the kinematic equations (10) and (11) lead to the following model:

Angular rate and translational acceleration measurements as controls:

$$\begin{aligned} \bar{p} &= \bar{s}_p p_m + \bar{b}_p + \bar{n}_p \\ \bar{q} &= \bar{s}_q q_m + \bar{b}_q + \bar{n}_q \\ \bar{r} &= \bar{s}_r r_m + \bar{b}_r + \bar{n}_r \end{aligned} \tag{14}$$

$$\begin{aligned} \bar{a}_{x_{cg}} &= \bar{s}_x a_{x_m} + \bar{b}_x + \bar{n}_x \\ \bar{a}_{y_{cg}} &= \bar{s}_y a_{y_m} + \bar{b}_y + \bar{n}_y \\ \bar{a}_{z_{cg}} &= \bar{s}_z a_{z_m} + \bar{b}_z + \bar{n}_z \end{aligned} \tag{15}$$

State-space equations:

$$\dot{\phi} = \bar{p} + \bar{q} \sin \phi \tan \theta + \bar{r} \cos \phi \tan \theta$$
$$\dot{\theta} = \bar{q} \cos \phi - \bar{r} \sin \phi \tag{16}$$
$$\dot{\psi} = (\bar{q} \sin \phi + \bar{r} \cos \phi)/\cos \theta$$

$$\begin{bmatrix} \ddot{x}_e \\ \ddot{y}_e \\ \ddot{h} \end{bmatrix} = T \begin{bmatrix} \bar{a}_{x_{cg}} \\ \bar{a}_{y_{cg}} \\ \bar{a}_{z_{cg}} \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \tag{17}$$

In addition to the rate gyros and translational accelerometers, an aircraft is often equipped with attitude sensors and a set of air data sensors. The latter include the airspeed V, the angle of attack α, and the sideslip angle β. Also, ground track data, i.e., slant range $\mathcal{R}$, bearing angle $\mathcal{B}$, and elevation angle $\mathcal{E}$, are measured during flight tests by a tracking radar. These measurements are related to the states of the preceding model by

Measurement equations:

$$V = \sqrt{u_V^2 + v_V^2 + w_V^2} \quad \alpha = \tan^{-1}(w_\alpha/u_\alpha) \quad \beta = \sin^{-1}(v_\beta/V_\beta) \tag{18}$$
$$\mathcal{R} = \sqrt{x_e^2 + y_e^2 + h^2} \quad \mathcal{B} = \tan^{-1}(y_e/x_e) \quad \mathcal{E} = \sin^{-1}(h/\mathcal{R})$$

where

$$u_\xi = u_{cg} + q\ell_{\xi_z} - r\ell_{\xi_y}$$
$$v_\xi = v_{cg} + r\ell_{\xi_x} - p\ell_{\xi_z}$$
$$w_\xi = w_{cg} - q\ell_{\xi_x} + p\ell_{\xi_y}$$
$$V_\xi = \sqrt{u_\xi^2 + v_\xi^2 + w_\xi^2}$$

and where ξ stands for V, α, or β. The measurement equations (18) are assumed to include scale factors, biases, and measurements noises, as presented in Eq. (12).

In the state-space equations (16) and (17), the measurement noises of the rate gyros and the translational accelerations, which are treated as controls, are now interpreted as the process noise inputs of the model. The random measurement errors present in the rest of the measurements are not correlated with these noises. Thus, the resulting model includes process noise inputs and measurement noises that are uncorrelated. The measurement noise of all of the sensors are also not correlated to the system states. These are important features of this formulation, allowing this model to be used with standard filtering and smoothing algorithms for the processing of the experimental data.

B. Kinematic Model Used in SMACK

A different model is used in the consistency check program called SMACK (smoothing of aircraft kinematics), developed at NASA Ames Research Center.[8,18] This is a nonlinear smoothing program that incorporates the nonlinear kinematics equations and includes the measurements errors, i.e., scale factors and biases, as additional constant states.

To simplify the nonlinear smoothing algorithm, a kinematic model is introduced where the state-space equations consist of pure integrators only, and all of the nonlinear kinematics are incorporated in the measurement equations. The state-space equations of this formulation are

$$\begin{bmatrix} \dddot{\phi} \\ \dddot{\theta} \\ \dddot{\psi} \end{bmatrix} = \begin{bmatrix} d_l \\ d_m \\ d_n \end{bmatrix} \qquad \begin{bmatrix} \dddot{x}_e \\ \dddot{y}_e \\ \dddot{h} \end{bmatrix} = \begin{bmatrix} d_x \\ d_y \\ d_h \end{bmatrix} \tag{19}$$

where d_l, d_m, d_n, d_x, d_y, and d_h are the unknown forcing functions, the "process noises" of the model. If sensor scale factors and bias errors have to be identified, additional state equations are introduced of the form

$$\dot{b} = 0 \tag{20}$$

Except for the attitude angles ϕ, θ, and ψ and position x_e, y_e, and h, all of the elements of the measurement model are nonlinear functions of the state variables. The angular velocities are

$$p = \dot{\phi} - \dot{\psi} \sin \theta$$
$$q = \dot{\theta} \cos \phi + \dot{\psi} \sin \phi \cos \theta \tag{21}$$
$$r = -\dot{\theta} \sin \phi + \dot{\psi} \cos \phi \cos \theta$$

Body axis components of the velocity u_a, v_a, and w_a, assuming calm atmosphere, and specific force $a_{x_{cg}}$, $a_{y_{cg}}$, and $a_{z_{cg}}$ are given by

$$\begin{bmatrix} u_a \\ v_a \\ w_a \end{bmatrix} = L \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{h} \end{bmatrix} \qquad \begin{bmatrix} a_{x_{cg}} \\ a_{y_{cg}} \\ a_{z_{cg}} \end{bmatrix} = L \begin{bmatrix} \ddot{x}_e \\ \ddot{y}_e \\ \ddot{h} + g \end{bmatrix} \tag{22}$$

where the transformation matrix L is the direction cosine matrix:

$$L = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & \sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & -\sin\phi\cos\theta \\ \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & -\cos\phi\cos\theta \end{bmatrix} \tag{23}$$

The air data measurements and the ground track data are given as before by

$$V = \sqrt{u_a^2 + v_a^2 + w_a^2} \quad \alpha = \tan^{-1}(w_a/u_a) \quad \beta = \sin^{-1}(v_a/V) \tag{24}$$
$$\mathcal{R} = \sqrt{x_e^2 + y_e^2 + h^2} \quad \mathcal{B} = \tan^{-1}(y_e/x_e) \quad \mathcal{E} = \sin^{-1}(h/\mathcal{R})$$

The effects of the sensors not being located at the center of gravity (c.g.) of the aircraft, that were presented in Eq. (18) and for simplicity are not presented here, are included in the SMACK model that also incorporates a simple wind model.[18]

A difficulty can arise in interpreting the process noise inputs d_l, d_m, d_n, d_x, d_y, d_h in Eqs. (19). Since they are equal to the *third derivatives* of the attitudes and positions, they involve the *first derivatives* of the specific moments and forces.

In deriving the smoother equations it is assumed that the process noise is uncorrelated with the system states. If the process noise is correlated with the system states, the preceding derivation is incorrect and may lead to biased smoothed results. It is hard to justify the lack of correlation of the inputs d_l, d_m, d_n, d_x, d_y, d_h with the states of the model given by Eqs. (19). For example, while using linear models of the aerodynamic forces and moments, the underlying assumption is that these forces and moments, and thus their derivatives, are *linearly* dependent on the aircraft body states through the stability derivatives.

These dependencies can be demonstrated using a one-degree-of-freedom example of the aircraft rolling motion. For this example the kinematic equations used in SMACK become

$$\dot{\phi} = p \tag{25}$$

$$\ddot{\phi} = \dot{p} \tag{26}$$

$$\dddot{\phi} = d_l \tag{27}$$

Assuming that the rolling moment acting on the aircraft is linear in the roll rate p and the aircraft aileron deflection δ_a, the roll dynamics equation is

$$\dot{p} = L_p p + L_{\delta_a} \delta_a \tag{28}$$

where L_p and L_{δ_a} are the stability and control derivatives. Differentiating Eq. (28) and using it again to substitute for $\dot{p}$ we get

$$\ddot{p} = L_p \dot{p} + L_{\delta_a} \dot{\delta}_a = L_p^2 p + L_{\delta_a}(L_p \delta_a + \dot{\delta}_a) \tag{29}$$

Substituting $\dddot{\phi}$ for $\ddot{p}$ and $\dot{\phi}$ for p, Eq. (29) becomes

$$\dddot{\phi} = L_p^2 \dot{\phi} + L_{\delta_a}(L_p \delta_a + \dot{\delta}_a) \tag{30}$$

By examining Eqs. (27) and (30) we conclude that

$$d_l = L_p^2 \dot{\phi} + L_{\delta_a}(L_p \delta_a + \dot{\delta}_a) \tag{31}$$

which means that the process noise d_l is linearly dependent on the system state $\dot{\phi}$ and the control inputs δ_a.

Such linear dependence, as demonstrated in the preceding example, means correlation of the forcing functions, the process noise in Eqs. (19), with the states. This correlation will cause errors in the smoothed estimates of the process noises and states. However, the good results obtained using SMACK[1,17,19] suggest that in the published examples correlations were small and their overall effect was not significant.

The difficulties described above are avoided using the formulation presented in the current paper, where the measurement noise of the rate gyros and translational accelerometers is treated as process noise. The assumption that the measurement noise is uncorrelated with the system states is easily justified allowing the model to be used with standard smoothing algorithms.

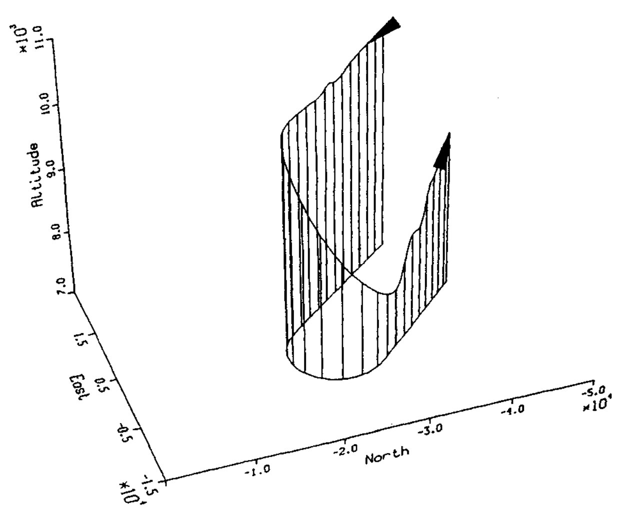

Fig. 1 VSRA flight path.

Table 1 VSRA measurement bias errors

Channel description	Biases
Roll rate, deg/s	0.018
Pitch rate, deg/s	0.023
Yaw rate, deg/s	0.014
Roll attitude, deg	−1.430
Yaw attitude, deg	−2.450
Longitudinal accelerometer, g	0.009
Lateral accelerometer, g	0.039
Vertical accelerometer, g	0.011

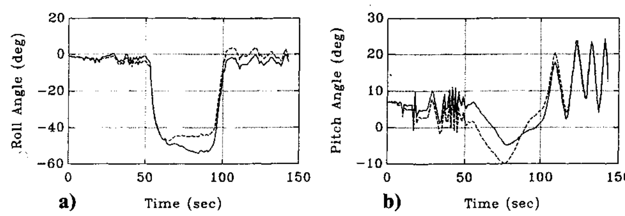

Fig. 2 VSRA initial data consistency check—attitude angles.

Another advantage of the suggested model is its lower dimension. The model described in Eqs. (16) and (17) has 9 states compared to the 18 states of the SMACK model, Eqs. (19). The former would have three additional states if rotational accelerations were required, leading to 12 states. This is a saving of 33% in the number of states, which is significant when the model is used for nonlinear smoothing. The saving is even larger when the additional states of the measurement scale factors and biases used in SMACK are considered. (See next section for further discussion on that issue.)

IV. Flight-Test Data Consistency Check

The flight-test data of the YAV-8B Harrier, the vertical and short takeoff and landing research aircraft (VSRA) of NASA Ames,[1] were checked for consistency using the smoothing identification algorithm with the nonlinear kinematic model presented earlier. The maneuver, shown in Fig. 1, was initiated from straight and level flight at an altitude of 10,500 ft and a heading of 300 deg, approximately. After 50 s of flight, the aircraft banked 60 deg to the left to turn 180 deg while losing about 2000 ft in altitude. At the end of the turn, it started to climb on a heading of approximately 120 deg. The maneuver lasted for 145 s. The aircraft states were measured at a sampling rate of 20 Hz.

Fifteen data channels were checked for consistency: three body axis components of the angular rate, three attitude angles, three body axis components of the specific forces sensed by accelerometers, and ground track data from two tracking radars, each measuring the slant range and bearing and elevation angles. The accelerometers were placed close to the c.g. of the aircraft, and thus, for simplicity, in this study it was assumed that they are located exactly at the c.g. The validity of this assumption was verified by repeating the checks with a model that included these displacements. The results obtained were similar to the ones presented here.

The flight-test data were initially checked for consistency by integrating the kinematic equations (16) and (17). These checks revealed some errors in the measurements. To demonstrate that, in Fig. 2 the measured attitude angles are compared with the results of integrating equations (16). The ground track data of one of the tracking radars are compared with the computed results from Eqs. (17) and (18) in Fig. 3. The differences seen in these comparisons indicate only small scale factors and bias errors in the measured angular rates and attitudes. The larger errors in the radar data are an indication of accelerometer measurement errors.

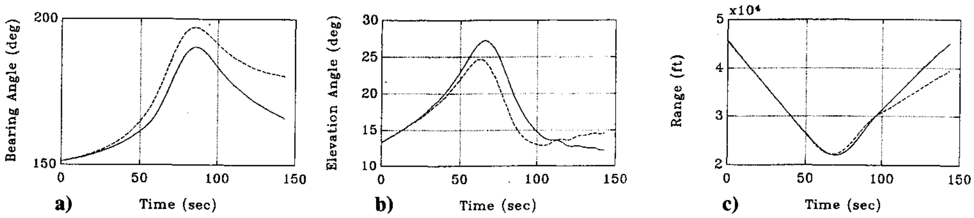

Fig. 3 VSRA initial data consistency check—tracking radar data.

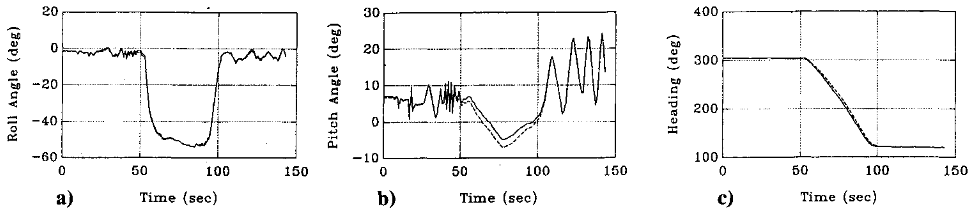

Fig. 4 VSRA consistency check—attitude angles.

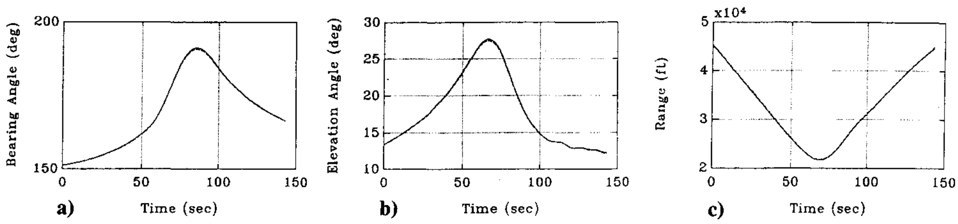

Fig. 5 VSRA consistency check—tracking radar 1.

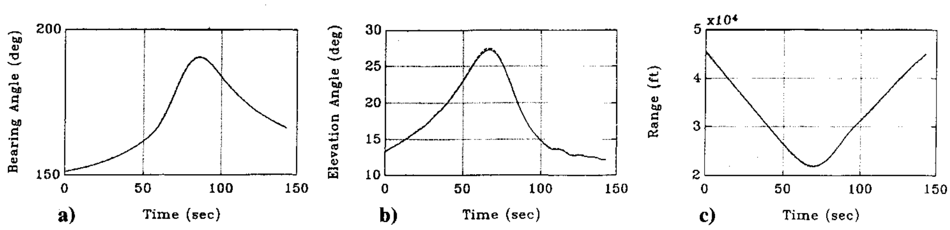

Fig. 6 VSRA consistency check—tracking radar 2.

The measurement errors were identified using the nonlinear kinematic model and the smoothing identification algorithm presented in this paper. The significant measurement errors were biases of some of the sensors and the scale factor of the roll attitude gyro. A total of nine measurement errors were identified. The identified values of the biases are presented in Table 1. The identified value of the roll attitude gyro scale factor is 1.018. These errors are not very large, but their effect on the consistency checks is significant.

The outputs of the model that includes the measurement errors are compared with the experimental data in Figs. 4–6. The close matches between the model outputs and the experimental data are clearly seen in those figures.

To identify the nine measurement errors, a nonlinear model with nine states, i.e., Eqs. (16) and (17), was used. That implies that the smoothing identification algorithm solves a ninth-order nonlinear smoothing problem at every iteration. By the approach presented in SMACK, the model would have had 15 states plus 9 additional constant states for the errors. The resulting nonlinear model would have had a total of 24 states. SMACK solves only one nonlinear smoothing problem, but in this case the dimension of it is drastically increased from 9 to 24. Solution of such a smoothing problem involves matrix equations of the order 24×24. In the smoothing identification algorithm presented here, this solution is replaced by a few solutions of a smoothing problem with the original model, i.e., solutions of 9×9 matrix equations. This reduction in the dimension of the problem has great computational and numerical advantages.

If a standard maximum likelihood procedure was used with the ninth-order model, to identify the 9 measurement errors and the 9 initial conditions, each iteration would involve the solution of 19 ninth-order extended Kalman filters, 1 nominal and 18 for the sensitivity function. This obviously involves many more computations than one solution of the smoothing problem solved in the current smoothing identification algorithm and that is its main advantage.

V. Conclusions

In this paper a parameter identification algorithm for nonlinear systems based on smoothing was presented. The algorithm smoothes the experimental data with different sets of system model parameters. One smoothing solution provides *all* of the information required to compute the gradients of the performance measure with respect to *any* number of system parameters. Computing the gradients from the smoothed data provides big computational savings compared with the standard maximum likelihood approach. The gradients are then used to update the parameters. The procedure is repeated until convergence is obtained.

The YAV-8B Harrier flight-test data were checked for consistency, based on the kinematic relationships between redundant sensors. A nonlinear kinematic model was presented. The use of the smoothing identification algorithm for nonlinear models was demonstrated in identifying the measurement scale factors and bias errors. The smoothed data that are corrected for the measurement errors can then be efficiently used for further analysis.

Appendix A: Smoothing Algorithm for Nonlinear Systems

In this Appendix a nonlinear smoothing algorithm is derived, following the approach presented in Refs. 8 and 20. The equations governing the nonlinear smoothing problem of discrete nonlinear systems are derived, followed by an algorithm that solves these equations. A discussion of hybrid nonlinear systems, in which the dynamics equations are continuous and the measurement equations are discrete, is presented at the end of this Appendix.

Discrete Nonlinear Smoother Algorithm

The fixed interval smoothing problem can be stated as follows: Given a discrete nonlinear system described by the state dynamics and measurement equations of the form

$$x(i+1) = f_d\left[x(i),u(i),w(i),i,\theta\right] \tag{A1}$$

$$z(i) = Eh_m\left[x(i),u(i),i,\theta\right] + b + v(i) \tag{A2}$$

and a sequence of experimental data $[z_m(i)]$, $i=1,\dots,N$, determine $x(0)$ and the sequence $[w(i)]$, $i=0,\dots,N-1$, that minimize the performance measure:

$$J = \tfrac{1}{2}\left[x(0)-x_0\right]^T P_0^{-1}\left[x(0)-x_0\right] + \frac{1}{2}\sum_{i=0}^{N-1}\left[w^T(i)Q^{-1}w(i) + v^T(i+1)R^{-1}v(i+1)\right] \tag{A3}$$

This is a nonlinear programming problem, since both the constraint equations (A1) and (A2) and the performance measure J are nonlinear in the unknowns of the problem. In addition to the usual quadratic nonlinearities in J, other nonlinearities are introduced through the last term of Eq. (A3), since

$$v(i+1) = z_m(i+1) - z(i+1) = z_m(i+1) - \left\{Eh_m\left[x(i+1),u(i+1),i+1,\theta\right] + b + v(i+1)\right\}$$

This nonlinear problem is solved by successively computing neighboring extremal paths that satisfy the constraint equations (A1) and (A2) until the minimum of the performance measure J is reached.[20] For that, a nominal trajectory that satisfies the constraint equations is computed. This can be done by specifying the initial condition $x(0)=x_0$ and the sequence $[w(i)]$, which in many cases can be assumed to be zero, and then propagating the system equations. A performance measure associated with this trajectory is computed using Eq. (A3). Based on the nominal trajectory, a neighboring trajectory is computed so that its performance measure is smaller than the previous one. The process is repeated until the minimum of J is obtained.

Variational techniques are used to compute neighboring trajectories from the nominal, by expanding the performance measure of Eq. (A3) to the second order and the constraint equations (A1) and (A2) to the first order.[20] The variation of the performance measure δJ due to the variations $[\delta x(i)]$ and $[\delta w(i)]$ is

$$\delta J = \left[x(0)-x_0\right]^T P_0^{-1}\delta x(0) + \tfrac{1}{2}\delta x(0)^T P_0^{-1}\delta x(0) + \sum_{i=0}^{N-1}\left[w^T(i)Q^{-1}\delta w(i) + \tfrac{1}{2}\delta w^T(i)Q^{-1}\delta w(i)\right] + \sum_{i=1}^{N}\left[-v^T(i)R^{-1}Eh_x(i)\delta x(i) + \tfrac{1}{2}\delta x^T(i)h_x^T(i)E^T R^{-1}Eh_x(i)\delta x(i)\right] \tag{A4}$$

The variation of the constraint equation (A1), which relates the variations $\delta x(i)$ to $\delta x(0)$ and $[\delta w(i)]$, is

$$\delta x(i+1) = f_x(i)\delta x(i) + f_w(i)\delta w(i), \quad \text{for } i=0,\dots,N-1 \tag{A5}$$

The gradient matrices $f_x(i)$, $f_w(i)$, and $h_x(i)$ in Eqs. (A4) and (A5) are defined as

$$f_x(i) = \left.\frac{\partial f_d(x,u,w,i,\theta)}{\partial x}\right|_{x(i),u(i),w(i)}$$

$$f_w(i) = \left.\frac{\partial f_d(x,u,w,i,\theta)}{\partial w}\right|_{x(i),u(i),w(i)}$$

$$h_x(i) = \left.\frac{\partial h_m(x,u,i,\theta)}{\partial x}\right|_{x(i),u(i)} \tag{A6}$$

and are evaluated along the nominal trajectory, given by the sequences $[x(i)]$, $[u(i)]$, and $[w(i)]$.

The objective is to compute a neighboring trajectory, defined by $\delta x(0)$ and $[\delta w(i)]$, so that the change in the performance measure δJ is as large a negative number as possible, while satisfying the constraint equation (A5). This defines an "accessory minimum" problem. Assuming that the partial derivatives $f_x(i)$, $f_w(i)$, and $h_x(i)$ are the same along the nominal and the neighboring trajectory (which mathematically follows from expanding the constraint equations to first order only thus assuming that second and higher order terms of this expansion are negligible), this minimization problem is similar to the smoothing problem of linear systems and can be solved using one of the existing smoothing algorithms.[11,12,20]

To show that similarity, the new minimization criterion δJ is modified by adding three terms:

$$\delta J_1 = \delta J + \tfrac{1}{2}\left[x(0)-x_0\right]^T P_0^{-1}\left[x(0)-x_0\right] + \frac{1}{2}\sum_{i=0}^{N-1}\left[w^T(i)Q^{-1}w(i) + v^T(i+1)R^{-1}v(i+1)\right] \tag{A7}$$

that are not a function of the minimization variables $\delta x(0)$ and $[\delta w(i)]$ and thus are constants in the accessory minimum problem. Substituting for δJ from Eq. (A4) and rearranging Eq. (A7), δJ_1 becomes

$$\delta J_1 = \tfrac{1}{2}\left[\delta x(0)+x(0)-x_0\right]^T P_0^{-1}\left[\delta x(0)+x(0)-x_0\right] + \frac{1}{2}\sum_{i=0}^{N-1}\left\{\left[w(i)+\delta w(i)\right]^T Q^{-1}\left[w(i)+\delta w(i)\right] + v_1^T(i+1)R^{-1}v_1(i+1)\right\} \tag{A8}$$

where

$$v_1(i) = z_m(i+1) - E\left\{h_x(i+1)\delta x(i) + h_m\left[x(i+1),u(i+1),i+1,\theta\right]\right\} - b$$

Minimizing δJ_1 with the constraint equation (A5) is a linear smoothing problem where $\delta x(0)$ are the unknown initial conditions and $[\delta w(i)]$ are the unknown inputs (process noise) to be computed. In this work, the linear backward information filter forward smoother algorithm[11,12] was used to solve this "accessory smoothing" problem. The solution is used to compute the neighboring trajectory that decreases the performance measure J of the nonlinear smoothing problem. The neighboring trajectories are computed sequentially until the changes of $\delta x(0)$ and $[\delta w(i)]$ are "sufficiently small." That implies that the change of δJ is "small" and the minimum of J is reached.

Nonlinear Backward Information Filter Forward Smoother

1) Using the initial conditions $x(0)$ and the sequence $[w(i)]$ obtained for the preceding iteration (or initial guesses), compute and store the nomimal trajectory, Eqs. (A1) and (A2); the performance measure J, Eq. (A3); and the gradient matrices $f_x(i)$, $f_w(i)$, and $h_x(i)$, Eqs. (A6).

2) For the backward information filter, set the final conditions $y_{N/N}=0$ and $S_{N/N}=0$. For $i=N,\ldots,1$, perform the measurement downdates:

$$y_{i/i}=y_{i/i+1}+h_x^T(i)E^TR^{-1}\{z(i)-Eh_m[x(i),u(i),i,\theta]-b\} \tag{A9}$$

$$S_{i/i}=S_{i/i+1}+h_x^T(i)E^TR^{-1}Eh_x(i) \tag{A10}$$

For $i=N-1,\ldots,0$, perform the time downdates:

$$K_B(i)=[Q^{-1}+f_w^T(i)S_{i+1/i+1}f_w(i)]^{-1}f_w^T(i)S_{i+1/i+1} \tag{A11}$$

$$w_B(i)=Qf_w^T(i)[I-f_w(i)K_B(i)]^Ty_{i+1/i+1} \tag{A12}$$

$$y_{i/i+1}=f_x^T(i)[I-f_w(i)K_B(i)]^T[y_{i+1/i+1}+S_{i+1/i+1}f_w(i)w(i)] \tag{A13}$$

$$S_{i/i+1}=f_x^T(i)[I-f_w(i)K_B(i)]^TS_{i+1/i+1}f_x(i) \tag{A14}$$

Store $y_{i/i}$, $w_B(i)$, $S_{i+1/i+1}$, and $K_B(i)$.

3) For the forward smoother, compute the initial conditions:

$$\delta x(0)=[S_{0/1}+P_0^{-1}]^{-1}\{y_{0/1}+P_0^{-1}[x_0-x(0)]\}$$

$$x_{\text{new}}(0)=x_{\text{old}}(0)+\delta x(0) \tag{A15}$$

For $i=0,\ldots,N-1$, compute

$$w_{\text{new}}(i)=w(i)+\delta w(i)$$

$$=w_B(i)-K_B(i)[f_x(i)\delta x(i)-f_w(i)w(i)] \tag{A16}$$

$$\delta x(i+1)=f_x(i)\delta x(i)+f_w(i)[w_{\text{new}}(i)-w(i)] \tag{A17}$$

$$\lambda(i)=S_{i/i}\delta x(i)-y_{i/i} \tag{A18}$$

4) Iterate until the changes in $x(0)$ and $w(i)$ are "sufficiently" small and the performance measure J is minimized.

Smoothing of Hybrid Nonlinear Systems

Today digital comptuers are widely used in flight-test data acquisition, which implies that the stored data are discrete time. Aircraft dynamics, however, are of a continuous nature and thus are described by a set of continuous differential equations. Smoothing of the data introduces a hybrid problem in which the state dynamics are continuous and the measurements are discrete time.

The models that are chosen to describe the system are usually valid only for a limited frequency range. For example, a rigid-body dynamics model of an aircraft can describe its motion up to a frequency of a few Hertz. To identify such models, the experimental data should be sampled "fast enough" to sufficiently describe the system dynamics. It is empirically known that the experimental data should be sampled at a frequency that is at least 10 times higher than the highest frequency of interest. Also, the measured data, which are continuous, have to be prefiltered to avoid the aliasing effects of sampling.[21]

Since smoothing using discrete experimental data is performed on a digital computer, the continuous part of the hybrid model must be approximated with a discrete model. For that, the control inputs between samples have to be approximated. A useful assumption is that the sampling is "fast enough" so that the continuous control inputs can be approximated with zero order holds (ZOH), i.e., the control inputs are assumed to be constant between samples. Using this approximation, the ZOH equivalent of the continuous *linear* dynamics model is computed by one of the well-known discretization algorithms.[21] If the system model is nonlinear, these discretization algorithms have to be modified.

As described in the preceding section, the nonlinear smoothing problem is solved by successively computing neighboring extremal paths. The trajectories for some assumed initial conditions and unknown inputs can be computed by numerical integration of the dynamics equations. Determining the neighboring trajectories involves a linear smoothing problem. In case of continuous systems, this smoothing problem is continuous and can be approximately solved by discretizing it. This affects only the first part of the smoothing algorithm presented earlier. This part of the algorithm, in which the nominal trajectory and the gradient matrices are computed, is changed to the following:

1) Using the initial conditions $x(t_0)$ and the sequence $[w(t_i)]$ obtained from the preceding iteration (or initial guesses), compute and store the nominal trajectory, by numerically integrating the dynamics equations

$$\dot{x}(t)=f_c[x(t),u(t),w(t),t,\theta] \tag{A19}$$

and determine the values of the states at the sampling points t_i, i.e.,

$$x(t_i)=\widehat{x(t_{i-1})}+\int_{t_{i-1}}^{t_i}f_c[x(t),u(t),w(t),t,\theta]\,\mathrm{d}t \tag{A20}$$

and then compute and store the performance measure J by numerically integrating

$$J=\tfrac{1}{2}[x(t_0)-x_0]^TP_0^{-1}[x(t_0)-x_0]+\frac{1}{2}\int_{t_0}^{tf}w^T(t)Q^{-1}w(t)\,\mathrm{d}t+\frac{1}{2}\sum_{i=1}^{N}\nu^T(t_i)R^{-1}\nu(t_i) \tag{A21}$$

where

$$\nu(t_i)=z_m(t_i)-\{Eh_m[x(t_i),u(t_i),\theta,t_i]+b\}$$

and then compute and store the *discrete* gradient matrices $f_x(i)$, $f_w(i)$, and $h_x(i)$ by linearizing the continuous nonlinear dynamics equations along the nominal trajectory and discretizing them:

$$f_x(i)=\text{DISC}\left[\left.\frac{\partial f_c(x,u,w,t,\theta)}{\partial x}\right|_{x(t_i),u(t_i),w(t_i),t_i}\right]$$

$$f_w(i)=\text{DISC}\left[\left.\frac{\partial f_c(x,u,w,t,\theta)}{\partial w}\right|_{x(t_i),u(t_i),w(t_i),t_i}\right]$$

$$h_x(i)=\text{DISC}\left[\left.\frac{\partial h_m(x,u,t,\theta)}{\partial x}\right|_{x(t_i),u(t_i),t_i}\right] \tag{A22}$$

where DISC[ ] is the discretization function.

The rest of the algorithm remains the same, as presented in the preceding section. Once the new initial conditions and the new unknown forcing functions are determined, a new trajectory can be computed. The procedure is repeated until convergence is obtained

Appendix B: Rank-Two Update Procedure

The rank-two update procedure is a technique for estimating the second-order gradient matrix, the Hessian, of the performance measure with respect to the parameters. In a broad view, this procedure is a numerical approximation of the Hessian from the values of the gradient evaluated with two sets of parameters at successive iterations. A derivation of the procedure outlined here can be found in Refs. 14 and 15.

Let θ_i and $\nabla_\theta J_i$ denote the parameters and the gradient of the performance measure with respect to those parameters at iteration i. The new values of the parameters are computed using a quasi-Newton procedure:

$$\theta_{i+1} = \theta_i - \alpha_i B_i^{-1} \nabla_\theta J_i \tag{B1}$$

where B_i is the estimate of the Hessian matrix for the parameters θ_i. The scalar α_i is the step size that leads to a minimum of the performance measure along the search direction given by $-B_i^{-1}\nabla_\theta J_i$. The gradient of the performance measure for the new values of the parameters θ_{i+1} is $\nabla_\theta J_{i+1}$. The change in the parameter and gradient vectors is defined as

$$p_i = \theta_{i+1} - \theta_i$$

$$q_i = \nabla_\theta J_{i+1} - \nabla_\theta J_i \tag{B2}$$

The estimate B_i of the Hessian matrix is updated by two rank-one matrices that are the outer products of the changes p_i and q_i. This provides an at most rank-two update of B_i and is given by

$$B_{i+1} = B_i + \frac{q_i q_i^T}{q_i^T p_i} - \frac{B_i p_i p_i^T B_i^T}{B_i^T p_i^T p_i} \tag{B3}$$

To show the rank-two property of this update better, Eq. (B3) can be arranged as

$$B_{i+1} = B_i + \frac{q_i q_i^T}{q_i^T p_i} + \alpha_i \frac{(\nabla_\theta J_i)(\nabla_\theta J_i)^T}{(\nabla_\theta J_i)^T p_i} \tag{B4}$$

where α_i is a scalar representing the step size of the quasi-Newton procedure described in Eq. (B1). From Eq. (B4) it can be concluded that, if the change in the gradient vector q_i is parallel to the gradient vector $\nabla_\theta J_i$, this update will be of rank one only.

The initial value of the Hessian matrix B_0 can be chosen as any symmetric positive definite matrix. The identity matrix is commonly used, causing the first parameter update to be in the steepest descent direction.

In this work, the rank-two procedure is used as a part of the quasi-Newton algorithm to update the system parameters, which uses the inverse of the Hessian matrix. Equation (B4) can be inverted to directly update the estimate of the inverse Hessian matrix H_i. The update is given by

$$H_{i+1} = H_i + \left(1 + \frac{q_i^T H_i q_i}{p_i^T q_i}\right) \frac{p_i p_i^T}{p_i^T q_i} - \frac{1}{p_i^T q_i}\left(p_i q_i^T H_i + H_i q_i p_i^T\right) \tag{B5}$$

This equation was incorporated into the identification procedure presented in this paper.

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